Final Report

JOINTLESS BRIDGES

by

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(The opinions, findings, and conclusions expressed in this report are those of the author and not necessarily those of the sponsoring agencies.)

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ABSTRACT

The results of this study are reported in two parts. The first deals with the various methods states are employing to reduce the number of joints in bridge decks. The most common method is the use of integral abutments, where the superstructure is joined to a flexible type of abutment. Typical designs of integral abutments are illustrated in the Interim Report on Jointless Bridges, dated November 1980.

The second part of the study deals with four new methods of reducing the number of joints in a bridge. These include the use of continuous jointless decks, flexible steel plate connectors between the superstructure and the piers, high cambering of the superstructure, and flexible piers. These four methods are analyzed mathematically, and from the analysis conclusions are drawn as to the feasibility of these methods.
INTRODUCTION

In June 1980, the working plan entitled "Jointless Bridges" was approved. The study described there was to be in two parts or phases. Phase I was to investigate existing methods of constructing bridge structures with the minimum number of joints so as to reduce long-term maintenance costs and possibly construction costs as well. Phase II was to investigate possible new methods of designing and constructing jointless bridges of lengths greater than those currently being built; generally bridges about 500 ft., or 150 m, long.

The results presented in this report follow the objectives of the working plan.

PHASE I

An interim report dealing with Phase I was prepared in January 1981 and sent to all interested parties, including the Bridge Research Advisory Committee and the Federal Highway Administration.

In Phase I, states known to have constructed some type of jointless bridge were contacted. These states were California, Colorado, Idaho, Iowa, Kansas, Missouri, Nebraska, North Dakota, South Dakota, Tennessee, Virginia, and Wisconsin. Information on the design, construction, and maintenance of their jointless bridges was obtained and summarized in the interim report. The following conclusions resulted from Phase I.

Of the states contacted, most use some type of integral abutments. In this type of abutment, a single row of piles is used to allow for flexing. A concrete pile cap is used to tie the piles to the bridge superstructure. Most designs provide a concrete sill extending from the end of the abutment to provide support for the approach pavement. The approach pavement should be of portland cement concrete, as bituminous pavements tend to crack
as a result of the movement of the bridge. The concrete pavement should be anchored to the abutment with reinforcing steel. This approach pavement should be designed in accordance with AASHTO specifications.

Other attachments extending from the approach to the bridge, as guardrails, should provide for some movement, as by using slotted bolt holes.

To avoid possible frost heaving of the abutment, water drains should be provided below the surface. Flexure stresses in the piles can be kept low, if necessary, by packing sand around the tops of the piles to allow for flexibility.

Skew angles over 30° can potentially cause problems in regard to cracking, torsion, or lateral slip. Until or unless further analysis is done on integral abutment bridges with large skews, they should not be used.

Some states have used integral abutments for as long as 20 years with good results. Different states have set different limits on the overall length of bridges with such abutments; however, in general, steel bridges up to about 300 ft. (90 m) and concrete bridges up to about 500 ft. (150 m) appear to perform satisfactorily. Several states, including Kansas and Tennessee, have integral abutment bridges of much greater length. Kansas has an 800 ft. (240 m) prestressed concrete bridge and Tennessee is building a prestressed concrete one 927 ft. (278 m) long; both have jointless decks and integral abutments.

The use of jointless decks and integral abutments has resulted in savings on the order of $10,000 for construction costs and even more in maintenance costs.

On the basis of this investigation, there is every reason to believe that if the integral abutment bridges selected for construction in Virginia are designed in accordance with these conclusions, the results will prove beneficial.

PHASE II

In the second part of the study on jointless bridges, several new methods relating to the elimination of joints were investigated. These new methods are categorized as Continuous Jointless Deck, Flexible Plate Connectors, High Deck Camber, and Flexible Piers. For each of these four methods, a simplified mathematical theory is presented, followed by a typical numerical example.
Continuous Jointless Deck

Continuously reinforced concrete highway pavements constructed without joints have been in existence for many years. In this type of pavement, the longitudinal movement of expansion and contraction is taken up by narrow, closely spaced, self-developed, transverse cracks in the concrete. The cracks are narrow enough to not cause any special maintenance problems. However, where the pavement meets a bridge structure, a joint has been introduced. It is the concept of the proposed continuous joint deck design that the continuously reinforced pavement extend across the bridge structure with no joint interruption.

Figure 1 illustrates how this is done. The figure shows a two-span bridge, but in principle the structure can have any number of spans, thereby making the system applicable for a bridge of any length. Should the highway pavement not be of continuously reinforced concrete, the bridge slab can be anchored as shown at position "B" in the figure. References 1 and 2 discuss anchorages that, when constructed properly, perform quite satisfactorily.

To allow for expansion and contraction of the bridge girders, they should be designed as simply supported members supported by flexible bearings as elastomeric pads. However, to take advantage of composite action between the girder and the deck slab, the central region of the girder, which carries the maximum bending moment, is compositely joined to the slab. The end regions of the girder are designed so as to allow slip at the interface between the girder and the slab. Several layers of plastic sheets at the interface are suggested to provide a slip plane. The bridge slab is expected to develop narrow, closely spaced, transverse cracks, but if the reinforcing steel is epoxy coated, as is current practice, no special problems are foreseen.

The two basic conditions of movement, namely expansion and contraction, were investigated analytically. Figure 1a illustrates, in an exaggerated form, the expansion behavior of a girder with slab in a bridge span as was shown in Figure 1. The continuously reinforced slab is assumed fixed between spans due to the nonmoveable nature of continuous pavements. The interface force, \( F \), in the composite region is assumed to act only at the ends of the composite region. Reference 3 shows this to be a close approximation. Any vertical forces resulting from incompatibility of vertical distortions between the slab and the girder are assumed to have a negligible effect on the system.

Based on these assumptions and that the steel and concrete remain elastic, the equations for the expansion conditions are derived as follows. The basic relations of stress and strain used are the well-established ones as found in reference 4.
Figure 1

CONTINUOUS JOINTLESS DECK DESIGN

A — Continuously reinforced concrete pavement and bridge deck
B — Deck anchorage, if deck joined to pavement type other than continuously reinforced concrete
C — Conventional abutment and pier with elastomeric bearings
D — Simply supported girder, composite with deck only in central region
   No bonding between deck and girder elsewhere
Figure 1a

A = Deck slab
B = Girder
C = Initial girder position at composite transition
D = Free expansion position of girder at transition
E = Restrained expansion position of girder and slab
F = Interface force between slab and girder

L = Girder span length
L1 = Composite zone length
L2 = Slip zone length

(L1, L1''/2, L1''/2)
For free expansion, referring to Figure 1a,

\[ L' = cT L, \]  

where \( c \) is the coefficient of expansion and \( T \) is the temperature change.

For the girder at the interface, considering both axial and bending strains,

\[ \frac{F L}{A G E_G} + \frac{F d^2 L}{E G G} = L'', \]  

where

- \( A_G \) is the cross-sectional area of the girder;
- \( E_G \) is the modulus of elasticity of the girder material;
- \( I_G \) is the moment of inertia of the girder cross section; and
- \( d_2 \) is the distance from the top flange of the girder to its own neutral axis.

For the slab in the composite region at the interface, considering both axial and bending strains,

\[ \frac{F_1 L}{A_R E_R} + \frac{F_1 d_1^2 L}{E_S S} = L' - L'', \]  

where

- \( F_1 \) is the tensile force in the slab in the composite region, assumed to be taken only by the longitudinal reinforcing;
- \( A_R \) is the cross-sectional area of the longitudinal reinforcing steel in the slab;
- \( E_R \) is the modulus of elasticity of the reinforcing bar material;
- \( E_S \) is the modulus of elasticity of the slab concrete;
$I_s$ is the moment of inertia of the slab cross section; and $d_1$ is assumed as one-half the slab thickness.

For the slab in the non-composite region at the interface, considering both axial and bending strains,

$$\frac{F_2 L_2}{A_s E_s} + \frac{F_2 d_1^2 L_2}{E_s I_s} = \frac{L_1' - L_1''}{2} \quad (4)$$

where $F_2$ is the compressive force in the slab in the non-composite region.

For equilibrium,

$$F_1 + F_2 = F \quad (5)$$
For convenience, equation 5 is substituted into equation 2 and rearranged as

\[ L_1'' = (F_1 + F_2) L_1 \left( \frac{1}{A_k G} + \frac{d_2^2}{E_{1} G} \right). \]  

(6)

Equation 3 is rewritten as

\[ L_1' - L_1'' = F_1 L_1 \left( \frac{1}{A_k E_R} + \frac{d_1^2}{E_{1} S} \right). \]  

(7)

Similarly, equation 4 is written as

\[ L_1' - L_1'' = 2F_2 L_2 \left( \frac{1}{A_k E} + \frac{d_2^2}{E_{1} S} \right). \]  

(8)

Equations 6, 7, and 8, along with equation 1, are left in this form to be solved numerically in an example problem to follow. Of special interest are the interface forces \( F_1, F_2, \) and \( F_3. \)

After the interface forces are determined, then the stresses generated by expansion in the girder can be computed for the slab and girder. These stresses, of course, must be superimposed onto all the other stresses in the span caused by direct thermal stress in the slab, dead load, live load, impact, and the like.

The equations for the critical stresses, \( f, \) due to expansion are as follows, considering both axial and bending behavior.

For the longitudinal reinforcing in the slab in the composite region,

\[ f_1 = \frac{F_1}{A_R} + \frac{F_1 d_1^2 E_R}{E_{1} S}. \]  

(9)

where a plus value is considered tension. For the top of the girder in the composite region,

\[ f_2 = -\frac{F}{A_G} - \frac{F d_2^2}{I_G}. \]  

(10)

where a minus value is considered compression. For the bottom
of the girder in the composite region,

$$f_3 = -\frac{F}{A_G} + \frac{F d_2 d_3}{I_G}, \quad (11)$$

where \(d_3\) is the distance from the bottom of the girder to its own neutral axis. For the bottom of the slab in the non-composite region,

$$f_4 = -\frac{F_2}{A_S} - \frac{F_2 d_1^2}{I_S}. \quad (12)$$

The equations given for the slab are only approximate, as the exact determination of stresses in reinforced concrete depends on the amount and location of the reinforcing steel and the quality of concrete.

Since no bond is developed between the slab and the girder in the non-composite region, and since the slab is under compression for this expansion condition, upward buckling of the slab over the length \(2L_2\) is a consideration. However, it is shown in reference 5 that for continuously reinforced concrete pavements, such uplift generally cannot occur.

The contraction condition is shown in Figure 1b. The assumptions of elasticity and the like hold for contraction as for expansion. Equations 1, 2, and 5 also remain valid. However, for the slab in the composite region at the interface,

$$\frac{F_1 L_1}{A_S E_S} + \frac{F_1 d_1^2 L}{E_S I_S} = L_1' - L_1''. \quad (13)$$

For the slab in the non-composite region at the interface,

$$\frac{F_2 L_2}{A_R E_R} + \frac{F_2 d_1^2 L_2}{E_S I_S} = \frac{L_1'' - L_1'}{2}. \quad (14)$$

After regrouping, the three basic equations for contraction are

$$L_1'' = (F_1 + F_2) L_1 \left(\frac{1}{A_G E_G} + \frac{d_2^2}{E_G I_G}\right). \quad (15)$$
\[ L_1' - L_1'' = F_1 L_1 \left( \frac{1}{A_S E_S} + \frac{d_1^2}{E_S I_S} \right), \text{ and} \]

\[ L_1' - L_1'' = 2F_2 L_2 \left( \frac{1}{A_R E_R} + \frac{d_1^2}{E_S I_S} \right). \] \hspace{1cm} (16) \hspace{1cm} (17)

The interface forces \( F, F_1, \) and \( F_2 \) can most easily be computed from these equations numerically. This is done in an example to follow. With the interface forces known, the critical stresses, \( f, \) in the slab and girder can be found for contraction in the girder as below.

For the bottom of the slab in the composite region,

\[ f_1 = -\frac{F_1}{A_S} - \frac{F_1 d_1^2}{I_S}. \] \hspace{1cm} (18)

For the top of the girder in the composite region,

\[ f_2 = \frac{F}{A_G} + \frac{F d_2^2}{I_G}. \] \hspace{1cm} (19)

For the bottom of the girder in the composite region,

\[ f_3 = \frac{F}{A_G} - \frac{F d_2 d_3}{I_G}. \] \hspace{1cm} (20)

For the longitudinal reinforcing in the slab in the non-composite region,

\[ f_4 = \frac{F_2}{A_R} + \frac{F_2 d_1^2 E_R}{E_S I_S}. \] \hspace{1cm} (21)

Numerical examples of the conditions of expansion and contraction for both steel and prestressed concrete girders are presented to illustrate the magnitude of forces and stresses likely to develop in the continuous jointless deck concept.
Consider first a steel girder system under an expansion condition. The following typical dimensions and material properties are assumed.

\[ \begin{align*}
L &= 1,200 \text{ in.} \ (30 \text{ m}) \\
L_1 &= L_2 = 400 \text{ in.} \ (10 \text{ m}) \\
c &= 6.5 \times 10^{-6} \text{ per degree F.} \ (11.7 \times 10^{-6} \text{ per degree C.}) \\
T &= 60^\circ \text{ F.} \ (33.3^\circ \text{ C.}) \\
A_G &= 127 \text{ in.}^2 \ (0.082 \text{ m}^2) \\
E_G &= 29 \times 10^6 \text{ lb./in.}^2 \ (200 \text{ GPa}) \\
I_G &= 58,000 \text{ in.}^4 \ (0.024 \text{ m}^4) \\
d_1 &= 3.5 \text{ in.} \ (0.089 \text{ m}) \\
d_2 = d_3 &= 18 \text{ in.} \ (0.457 \text{ m}) \\
A_R &= 3 \text{ in.}^2 \ (0.00194 \text{ m}^2) \\
E_R &= 29 \times 10^6 \text{ lb./in.}^2 \ (200 \text{ GPa}) \\
A_S &= 504 \text{ in.}^2 \ (0.325 \text{ m}^2) \\
E_S &= 4 \times 10^6 \text{ lb./in.}^2 \ (27.6 \text{ GPa}) \\
I_S &= 2,058 \text{ in.}^4 \ (0.00085 \text{ m}^4)
\end{align*} \]

Substituting the appropriate numerical values into equations 1, 6, 7, and 8 and solving these algebraic equations simultaneously, the following values are obtained:

\[ \begin{align*}
F &= 111,721 \text{ lb.} \ (496.9 \text{ kN}) \\
F_1 &= 26,025 \text{ lb.} \ (115.7 \text{ kN}) \\
F_2 &= 85,696 \text{ lb.} \ (381.2 \text{ kN})
\end{align*} \]
Using equations 9, 10, 11, and 12, stresses \( f_1, f_2, f_3, \) and \( f_4 \) for the condition of expansion can be obtained. For this example, they are:

\[
\begin{align*}
  f_1 &= 9,914 \text{ lb./in.}^2 \text{ tension (68.31 MPa)} \\
  f_2 &= 1,503 \text{ lb./in.}^2 \text{ compression (10.36 MPa)} \\
  f_3 &= 255 \text{ lb./in.}^2 \text{ compression (1.76 MPa)} \\
  f_4 &= 680 \text{ lb./in.}^2 \text{ compression (4.69 MPa)}
\end{align*}
\]

Of these forces and stresses, the 111,721-lb. (496.9-kN) force in the shear connectors is the only one that warrants special treatment. Being rather large, this shear force should be spread over some reasonable distance on the girder to reduce the stress concentration effect. It is to be noted that although the value for \( f_1 \) appears somewhat large, this stress is in the longitudinal direction, whereas stresses in the slab generated by dead and live loads are in the transverse direction and are carried by a different set of reinforcing steel.

Next, consider this same steel girder system under a contraction condition. The numerical values as for the expansion condition are again assumed.

Employing equations 1, 15, 16, and 17, the contraction values for \( F, F_1, \) and \( F_2 \) are obtained as follows.

\[
\begin{align*}
  F &= 150,197 \text{ lb. (668 kN)} \\
  F_1 &= 139,550 \text{ lb. (620.7 kN)} \\
  F_2 &= 10,647 \text{ lb. (47.4 kN)}
\end{align*}
\]

Then, with equations 18, 19, 20, and 21, the values of \( f_1, f_2, f_3 \) and \( f_4 \) for contraction are:

\[
\begin{align*}
  f_1 &= 1,107 \text{ lb./in.}^2 \text{ compression (7.63 MPa)} \\
  f_2 &= 2,021 \text{ lb./in.}^2 \text{ tension (13.92 MPa)} \\
  f_3 &= 343 \text{ lb./in.}^2 \text{ tension (2.36 MPa)} \\
  f_4 &= 4,056 \text{ lb./in.}^2 \text{ tension (27.95 MPa)}
\end{align*}
\]
Forces and stresses for contraction are seen as being of the same order of magnitude as for expansion for the same degree of temperature change. Comments pertaining to expansion stresses in the preceding example, therefore, apply as well to these stresses.

As another example, consider the same basic problem except that a prestressed concrete girder is substituted for the steel girder. All the numerical values are the same as for the preceding example except for the following.

- $c = 6 \times 10^{-6}$ per degree F. $(10.8 \times 10^{-6}$ per degree C.)
- $A_G = 736 \text{ in.}^2 \ (0.475 \text{ m}^2)$
- $E_G = 4 \times 10^6 \text{ lb./in.}^2 \ (27.6 \text{ GPa})$
- $I_G = 508,000 \text{ in.}^4 \ (0.210 \text{ m}^4)$
- $d_2 = d_3 = 24 \text{ in.} \ (0.61 \text{ m})$

For expansion, the forces $F, F_1,$ and $F_2,$ obtained by the simultaneous solution of equations 1, 6, 7, and 8, are as follows:

- $F = 98,111 \text{ lb.} \ (436.4 \text{ kN})$
- $F_1 = 23,021 \text{ lb.} \ (102.4 \text{ kN})$
- $F_2 = 75,089 \text{ lb.} \ (334.0 \text{ kN})$

Then, by use of equations 9, 10, 11, and 12, the stresses $f_1, f_2, f_3,$ and $f_4$ are:

- $f_1 = 8,770 \text{ lb./in.}^2 \ \text{tension} \ (60.42 \text{ MPa})$
- $f_2 = 244 \text{ lb./in.}^2 \ \text{compression} \ (1.68 \text{ MPa})$
- $f_3 = 22 \text{ lb./in.}^2 \ \text{compression} \ (0.15 \text{ MPa})$
- $f_4 = 595 \text{ lb./in.}^2 \ \text{compression} \ (4.10 \text{ MPa})$

It is seen from these values that the forces and stresses in a concrete girder system are somewhat less than those in a steel girder system for the same basic conditions of length and temperature.
For the contraction condition of a concrete girder system, equations 1, 15, 16, and 17 are used to find the following forces $F$, $F_1$, and $F_2$:

$$F = 149,481 \text{ lb. (664.9 kN)}$$
$$F_1 = 135,640 \text{ lb. (603.3 kN)}$$
$$F_2 = 13,841 \text{ lb. (61.6 kN)}$$

As obtained from equations 18, 19, 20, and 21, the stresses for the contraction condition $f_1$, $f_2$, $f_3$, and $f_4$ are:

$$f_1 = 1,076 \text{ lb./in.}^2 \text{ compression (7.41 MPa)}$$
$$f_2 = 373 \text{ lb./in.}^2 \text{ tension (2.57 MPa)}$$
$$f_3 = 33 \text{ lb./in.}^2 \text{ tension (0.23 MPa)}$$
$$f_4 = 5,272 \text{ lb./in.}^2 \text{ tension (36.32 MPa)}$$

As in the steel system, the critical aspect appears to be the force, $F$, on the shear connections, which would require the spreading of this force over some reasonable distance along the girder flange.

Flexible Plate Connectors

Many bearing devices have been used in bridge construction to take up the longitudinal movement in the superstructure caused by temperature changes and similar factors. Most of these require periodic maintenance. In an attempt to find an adjustable bearing that requires little or no maintenance, the flex-plate connector illustrated in Figure 2 was investigated. In concept, this plate of stainless steel would be permanently fixed to both the pier and the superstructure girder and would carry the vertical loads as well as adjust for longitudinal movement.

The basic elastic theory of behavior of this kind of plate as developed by classical mechanics is presented in reference 4. The relevant equations as adapted to this flex-plate problem shown in Figures 2a and 2b are presented below.

$$d = c TL,$$ 
(22)
where
c is the coefficient of expansion;
T is the temperature change; and
L is the span length.

\[ \Delta = \frac{PH^3}{12ET}, \]  

(23)

where

P is the horizontal displacing force;
H is the height of the plate;
E is the modulus of elasticity of the plate; and
I is the moment of inertia of the plate cross section.

\[ I = \frac{bt^2}{6}, \]  

(24)

where b is the width of the plate and t is the thickness of the plate.

\[ M = \frac{PH}{2} \quad \text{and} \quad M' = Qd, \]  

(25)

(26)

where Q is the vertical force on the plate.

A third moment, \( M'' \), could be generated in the plate if the girder rotates an angle \( N \) from its original position at the support.

\[ M'' = \frac{EIN}{H}, \]  

(27)

where the angle \( N \) is in radians.
Figure 2
FLEX-PLATE DESIGN

A = Girder
B = Stainless steel plate or Plates
C = Deflected position of plate
D = Pier or abutment
E = Girder movement
Figure 2a  FLEX-PLATE CONDITIONS
2a - Forces generated by displacement d

Figure 2b
2b - Forces generated by load Q
The combined maximum stress, \( f \), in the plate is then

\[
f = \frac{P}{bt} \pm \frac{Mt}{2I} \pm \frac{M't}{2I} \pm \frac{M''t}{2I},
\]

(28)

where positive values are tension stresses and negative values are compressive stresses.

Buckling of the plate is another consideration. Assuming elastic behavior, the buckling load, \( Q' \), is given by Euler's equation as

\[
Q' = \frac{4H^2EI}{H^2}.
\]

(29)

A numerical example is presented to indicate the magnitude of stresses induced in a trial flex-plate design. The following values are assumed.

- \( c = 6.5 \times 10^{-6} \) per degree F. (11.7 \times 10^{-6} per degree C.)
- \( T = 60^\circ \) F. (33.3^\circ \) C.)
- \( L = 1,200 \) in. (30 m)
- \( H = 12 \) in. (0.3 m)
- \( E = 29 \times 10^6 \) lb./in.\(^2\) (200 GPa)
- \( b = 12 \) in. (0.3 m)
- \( t = 0.25 \) in. (0.006 m)
- \( Q = 135,000 \) lb. (600 kN)
- \( N = 0 \)

By application of equations 22 through 28, the maximum stress in the plate is computed as 664 k/in.\(^2\) (4,437 MPa). The buckling load is 992 kips (4,412 kN) as determined from equation 29. Although there is no danger of buckling, the plate is greatly overstressed and is, therefore, an unsatisfactory design.

The maximum stress in the plate could be reduced to 322 k/in.\(^2\) (2,218 MPa) if three such plates were sandwiched together; however, even this value is too large.
If another set of three plates is located at the bottom of the pier as well as at the top; the maximum stress can be reduced to about 171 k/in.\(^2\) (1,178 MPa) for a very tall pier. For short piers, the stress level lies between 171 k/in.\(^2\) (1,178 MPa) and 322 k/in.\(^2\) (2,218 MPa), depending on the height of the pier.

However, considering fatigue failure at high stress levels, as well as high elastic stresses, this proposed concept of using flexible plate connectors, although possible, does not appear to be a practically feasible solution.

**High Deck Camber**

The principle of high deck camber design is illustrated in Figure 3. By humping the horizontal spanning structure, the actual length along the humped configuration is made greater than the straight-line length between supports. Thus, in concept, the camber would decrease for contraction conditions and increase for expansion conditions, assuming the structure fixed at the supports. This concept is widely used to accommodate expansion and contraction in the installation of long pipe lines. The analysis that follows is to determine if this principle is valid for long bridge structures.

To simplify the analysis, a linearized camber is assumed as shown in Figure 3a. Referring to Figure 3b, the value for the restraining force, \(P\), is obtained by equating the external work on the system to the internal elastic strain energy. This method of analysis is described in reference 4. Second order effects are neglected.

External work = Direct strain energy + Bending strain energy (30)

\[
\frac{Pd}{2} = \frac{P^2L}{2AE} + PhL
\]

where

\(A\) is the girder cross-sectional area;

\(E\) is the modulus of elasticity of the material; and

\(I\) is the girder moment of inertia.
Figure 3
HIGH CAMBER DESIGN

A = Spanning superstructure (shown with exaggerated camber)
B = Camber after longitudinal shortening
C = Pier or abutment
Figure 3a

LINEARIZED CAMBER

$h = \text{Rise}$

$L = \text{Span length}$
Figure 3b
EXPANSION CONDITION

- \( h \) = Free expansion
- \( h' \) = Increase in camber
- \( P \) = Restraining force
- \( d \) = Distance
- \( L/2 \) = Half length
Free expansion, \( d \), is given by the equation

\[
d = c \, TL, \tag{32}
\]

where \( c \) is the coefficient of expansion and \( T \) is the change in temperature. By substituting equation 32 into equation 31, the equation for the restraining force \( P \) is obtained as

\[
P = \frac{A \, (2c \, EIT-h)}{2T}. \tag{33}
\]

The maximum bending moment, \( M \), in the cambered girder, neglecting the change in \( h \), is

\[
M = Ph. \tag{34}
\]

The change in camber, \( h' \), can be found by the use of the conjugate beam method of determining deflections. Such a conjugate beam diagram is shown in Figure 3c. In the conjugate beam method, the conjugate beam bending moment is equal to the deflection in the original beam. Therefore, from Figure 3c, the bending moment in the center of the beam is determined and equated to \( h' \):

\[
h' = \frac{PhL^2}{2EI}. \tag{35}
\]

A somewhat more exact value for the maximum bending moment in the original cambered girder is

\[
M = P \, (h \pm h'), \tag{36}
\]

where the positive value for \( h' \) is for expansion and the negative value is for contraction.

Considering both axial and bending stresses, the maximum stresses induced in the girder \( f \) due to expansion are

\[
f = -\frac{P}{A} \pm \frac{M}{S}, \tag{37}
\]

where \( S \) is the section modulus of the cross section referred to the bottom of the girder for compression (negative value) and top of the girder for tension (positive value).
For contraction conditions, the maximum tensile stress, \( f \), is computed by

\[
f = \frac{P + M}{A + S},
\]

(38)

where \( S \) is the section modulus of the cross section referred to the bottom of the girder for tension (positive value) and top of the girder for compression (negative value).

Considering the action of normal vertical dead and live loads, the critical condition is that of contraction inducing an additional tensile stress in the bottom of the girder at mid-span. The numerical example to follow illustrates the magnitude of this critical tensile stress due to contraction.

The following values are assumed for a typical continuous bridge with a steel girder and composite concrete deck.

\[
\begin{align*}
L &= 2,400 \text{ in. (60 m)} \\
h &= 24 \text{ in. (0.61 m)} \\
E &= 29 \times 10^6 \text{ lb./in.}^2 (200 \text{ GPa}) \\
c &= 6.5 \times 10^{-6} \text{ per degree F. (11.7} \times 10^{-6} \text{ per degree C.)} \\
I &= 137,000 \text{ in.}^4 (0.057 \text{ m}^4) \\
A &= 200 \text{ in.}^2 (0.129 \text{ m}^2) \\
S &= 2,600 \text{ in.}^3 (0.042 \text{ m}^3) \\
T &= 60^\circ \text{ F. (33.3}^\circ \text{ C.)}
\end{align*}
\]

From equation 33 the restraining tensile force, \( P \), is 2,262 kips (10.06 MN). From equation 35 the decrease in camber, \( h' \), is 6.56 in. (0.17 m). From equation 38, the maximum tensile stress (located in the bottom of the girder) is 26,483 lb./in.\(^2\) (182.5 MPa). Equation 36 is used for the computation of \( M \).

Several conclusions may be drawn from this example.

1. The restraining force, \( P \), is very large and not sensitive to small values of camber. For all practical purposes, the restraining force for either expansion or contraction is as if the girder had no camber.
2. The amount of camber changes a significant amount (27.3% in this case) under the action of expansion and contraction.

3. The critical tensile stress for contraction alone is rather large. When combined with dead and live load tensile stresses, contraction stresses could induce an over-stress condition, unless high strength steel is used.

4. In view of the impaired riding qualities of the bridge deck caused by high camber (2% grade in this example), coupled with high induced stresses, this method, while technically possible, is not practically feasible.

Flexible Piers

In many situations, piers in themselves can be used to either restrain longitudinal length changes or accommodate length changes, thereby reducing the need for deck joints. It is stated in the Interim Report (pg. 11) that the state of Tennessee has constructed a continuous box girder bridge 2,700 ft. (810 m) long with joints at the abutments only. The girders are dowled to the concrete piers, spaced approximately 100 ft. (30 m) apart. Field observation has shown that the end movement at the joints is only a fraction of that expected by free expansion. The explanation lies in the restraint offered by the relatively low piers.

Whereas it is possible to restrain the longitudinal movement (in whole or in part), it is believed more economical to accommodate such movement by allowing the piers to flex as seen in Figure 4. This is particularly desirable and easy to do if the piers are tall. Figure 4a illustrates how the superstructure and concrete piers could be joined for either concrete or steel girders. The elastomeric pads are there only to absorb rotational movements.

Using basic beam theory (reference 4) in relation to Figure 4b, the deflecting force, $P$, is given as below. For simplicity, elastic homogeneous theory is used.

\[ P = \frac{3EId}{H^3}, \]  

(39)

where $E$ is the modulus of elasticity of the pier material and $I$ is the section modulus of the pier cross section.
Figure 4

FLEX-PIER DESIGN

A - Superstructure, fixed to pier
B - Flexible pier
C - Deflected position of pier
D - Girder movement
TYPICAL PIER CONNECTION DETAILS

A = Deck slab
B = Concrete girder
C = Steel girder
D = Elastomeric bearing pad
E = Anchor bolt
F = Reinforced concrete pier

Figure 4a
Figure 4b

FLEX-PIER FORCES

d = Displacement
H = Pier height
P = Displacement force
The deflected distance $d$ is computed as

$$d = c TL,$$  \hspace{1cm} (40)

where

- $c$ is the coefficient of expansion of the girder material;
- $T$ is the temperature change; and
- $L$ is the bridge span from a fixed end.

The maximum bending moment, $M$, is determined as

$$M = PH.$$  \hspace{1cm} (41)

The maximum bending stress is then obtained as

$$f = \frac{M}{S},$$  \hspace{1cm} (42)

where $S$ is the section modulus of the pier cross section at its base.

Consider the following situation as an example.

$H = 1,200$ in. (30 m)

$E = 4 \times 10^6$ lb./in.$^2$ (27.6 GPa)

$I = 4,478,976$ in.$^4$ (1.85 m$^4$) based on a pier 6 ft. by 12 ft. (1.8 m by 3.6 m) in cross section

$S = 124,416$ in.$^3$ (2.03 m$^3$)

$c = 6.5 \times 10^{-6}$ per degree F. ($11.7 \times 10^{-6}$ per degree C.)

$T = 60^\circ$ F. (33.3$^\circ$ C.)

$L = 2,400$ in. (60 m)

From equations 39 and 40, the displacing force, $P$, is 29,100 lb. (129.4 kN). Then from equations 41 and 42, the maximum bending stress, $f$, is 281 lb./in.$^2$ (1.76 MPa).

It is seen that the stress level is very low for this 100 ft. (30 m) tall pier. For comparison, if a 50 ft. (15 m) high pier of the same cross section is analyzed, the maximum stress is determined to be 1,123 lb./in.$^2$ (7.74 MPa).
From these data, it is concluded that tall flexible piers can easily be used to absorb longitudinal expansion or contraction. As piers become shorter, stress levels due to flexing can be expected to increase. At such conditions, pier flexing is a less desirable method of accommodating movement, although the use of prestressed concrete piers could provide both greater flexibility and strength to short piers.

CONCLUSIONS

Because this study has two phases, the conclusions are also presented in two parts. The conclusions for the first phase appear in the section of this report under Phase I. The conclusions that appear here pertain primarily to Phase II.

In Phase II, four methods of reducing the need for joints were analyzed mathematically. Based on the analysis, the following conclusions are drawn concerning their feasibility.

Continuous Jointless Deck

This method would allow bridges of any length to be constructed with absolutely no joints in the deck. In addition, there are no joints between the bridge deck and the roadway pavement. Stresses induced in the slab and girder in the longitudinal direction are not excessively high, and with the proper amount and type of material (steel or concrete), these stresses can be easily accommodated.

The large forces on the connectors between the girder and the slab could present a problem, however. The theory developed assumes that the interface force is concentrated at one point, whereas due to stress redistribution, this force may be spread out over some distance. With the redistribution, the magnitude of the stress level in the connectors is considerably decreased. Further investigation of this redistribution is desirable. Some suggestions for future research on this matter are outlined in this report under Recommendations.

Other issues, such as the nature of transverse cracking induced in the slab and the behavior of the slip plane between the slab and the girder, also warrant attention. These are not seen as problems but as features to be noted when and if such a continuous jointless deck is constructed.
Flexible Plate Connectors

The concept of using flex-plates to permanently join the superstructure to the piers is an interesting one in that the elements are fixed through continuity of the structure, yet are moveable. However, given the characteristics of present-day steels, the stress levels in flexible plate connectors are unacceptably high.

This method is, therefore, seen as not being practically feasible for general use.

High Deck Camber

Theoretically, large cambering is a valid method of absorbing longitudinal movement. However, as analysis shows, the stresses developed in a practical bridge system are excessively high. In addition, large cambers, especially for multi-span bridges, adversely affect the riding quality. Thus, for commonly used bridges of short or moderate span, this method of eliminating joints is undesirable. For long bridges, as suspension or cable-stayed types, high cambering may have application, although no analysis of such long structures was developed in this study.

Flexible Piers

Tall flexible piers work automatically as devices to relieve stresses caused by longitudinal length changes. Generally, such action is not taken advantage of by bridge designers. Analysis shows that tall piers of contemporary design do flex, and can absorb a reasonable amount of motion without overstress.

A bridge system utilizing flexible piers has the potential of eliminating all but one joint in the deck and all but one moveable bearing. Under the circumstances discussed in Phase I, the use of integral abutments could even eliminate the need for this one joint and moveable bearing.

Generalizations concerning the length of bridge for which this method is valid cannot be made because of the many variables involved, but it appears reasonable that if integral abutments are used with flexible piers, the limits imposed on integral abutment bridges hold also for flexible piers.
General Statement

As a general conclusion relevant to all types of systems considered for jointless bridges, three types are worthy of serious consideration. These are integral abutments, continuous jointless decks, and flexible piers. Integral abutments have been used successfully for many years in many states, although not in Virginia. The use of continuous jointless decks is a new concept that may allow bridges of any length to be constructed without joints in the deck. Further research is needed on it before implementation. Flexible pier design is a straightforward method that has application in those special situations where the piers are tall, as in a valley or deep water crossing.

Finally, this study has shown that methods are possible to eliminate completely or reduce the number of joints needed in a bridge, and thereby decrease long-term maintenance costs and possibly construction costs as well.

RECOMMENDATIONS

1. It is recommended that Virginia proceed with the incorporation of integral abutments in the design of new bridges, using the information developed in this study for guidance. After the construction of several such bridges, their general performance should be monitored to determine if they are functioning as expected.

2. It is recommended that Virginia consider the utilization of pier flexibility to reduce or eliminate the number of joints in a bridge. Appropriate candidates for such a design are bridges with tall, slim concrete piers.

3. It is recommended that further research be done on the proposed continuous jointless deck concept. Of particular interest is the behavior of the shear connectors under the special conditions imposed by the design. Two research methods are possible. The first is a rigorous mathematical analysis using the finite element method. This would require extensive use of the digital computer and would be based on assumptions of material behavior. A second method, and the one preferred, is experimental and does not rely on material assumptions.

To keep the cost of testing under the second method to a minimum, a one-quarter-scale model of a bridge span is suggested. A 100-ft. (30-m) span prototype structure would
then be reduced to 25 ft. (7.5 m). Only a single girder, with slab, needs to be constructed. The girder would have simply supported ends and the deck slab would be fixed against longitudinal movement at its ends. This could be done by using anchors, walls, or a U frame. A steel girder is suggested as it is easy to heat, as with thermally controlled heating wires taped to the girder. Insulation around the girder is desirable to retain the heat. Instrumentation would consist of thermocouples along the girder and in the slab, and electrical resistance strain gages in the reinforcing steel and along the girder. Closely spaced strain gages should be placed under the top flange of the girder in the region of the shear connectors to determine the stress transfer behavior in the composite region.

In this same test apparatus, vertical loads as well as thermally induced loads can be applied to observe their interaction effects.

Cooling of the girder is difficult to do experimentally, but may not be necessary as the shear transfer mechanism is essentially the same for expansion or contraction, except for the reversal in the direction of stresses.

Assuming that laboratory tests prove satisfactory, the construction of a full-scale bridge should then be considered.
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REFERENCES CITED


SELECTED REFERENCES


