FINAL REPORT

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OF BRIDGE RESPONSE
USING ACCELERATION DATA

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This research evaluated various methods to calculate the displacement response of a bridge using measured acceleration data. Methods included the use of integration schemes and the correction algorithms necessary for accurately determining displacements. Corrections are needed since any recorded signal contains error and the initial conditions of a structural system are not always known. Different numerical integration schemes and correction algorithms were applied to acceleration signals developed analytically, and these methods were then evaluated using acceleration signals recorded from laboratory tests. Finally, the methods were applied to acceleration data recorded from a field test of a highway bridge. In each case, the calculated displacement response was compared to the measured or exact displacement response to provide a basis for comparison. From the insight gained in this investigation, recommendations were made concerning the accurate determination of the displacement response of a bridge using accelerometer data.
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(The opinions, findings, and conclusions expressed in this report are those of the authors and not necessarily those of the sponsoring agencies.)

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ABSTRACT

Knowledge of the actual displacement response of a bridge subjected to random traffic loading is useful in evaluating bridge performance and serviceability. However, mounting displacement transducers is difficult, and the feasibility of and cost associated with such instrumentation are often problems.

This research evaluated various methods to calculate the displacement response of a bridge using measured acceleration data. Methods included the use of integration schemes and the correction algorithms necessary for accurately determining displacements. Corrections are needed since any recorded signal contains error and the initial conditions of a structural system are not always known. Different numerical integration schemes and correction algorithms were applied to acceleration signals developed analytically, and these methods were then evaluated using acceleration signals recorded from laboratory tests. Finally, the methods were applied to acceleration data recorded from a field test of a highway bridge. In each case, the calculated displacement response was compared to the measured or exact displacement response to provide a basis for comparison. From the insight gained in this investigation, recommendations were made concerning the accurate determination of the displacement response of a bridge using accelerometer data.
INTRODUCTION

Dynamic response is an important aspect of bridge behavior, and dynamic analysis can be an important tool for identifying the severity of damage or deterioration in bridges. This type of analysis also provides a useful procedure for understanding the fundamental response of a bridge structure and the effect of various structural modifications on its subsequent behavior and safety. To provide for the reliable assessment of bridge behavior and ensure continued integrity of the structure, systematic ways of determining the vibration characteristics of a bridge and its interaction with traffic are essential.

Currently, there are two common procedures for determining the dynamic response of a bridge. The most effective is to measure the actual dynamic response experimentally. Although this approach provides accurate response data, field testing can be time-consuming and expensive. The second method is to construct a computer model of the bridge and analyze it using a finite element code. With any finite element approach, however, approximations are made, which necessarily affect the results. The two approaches are commonly used together to obtain a more comprehensive evaluation of bridge behavior.

Transducers used in field tests are frequently chosen based on convenience and the cost of installation. Accelerometers are usually the transducer selected because they are easy to install and have a relatively low cost. The acceleration response can provide valuable information on the bending and torsional modes of the bridge and associated natural frequencies. Also, the damping and impact factors of the bridge may be evaluated by analyzing the acceleration response. However, the displacement and velocity responses are also frequently desired.

Measuring the vertical displacements of bridges experimentally is not a trivial matter. Displacement transducers measure relative displacements and thus need a stationary reference
Establishing such a base requires access underneath a bridge, which is not always possible. Even where access is possible, the height of some bridges makes the installation of these transducers impractical. Therefore, if obtaining the displacement response of a typical bridge experimentally is desired, an accurate determination of the displacements from measured accelerations is exceedingly desirable.

Previous research has involved the use of both accelerometers and displacement transducers, usually linear variable displacement transducers (LVDTs). In a dynamic field test of the Route 265 Dan River Bridge in Virginia (Wolek, 1992), an array of accelerometers and one LVDT were used to record response data. The acceleration data were integrated twice and compared to the corresponding measured displacements. These calculated and measured displacements compared reasonably well for very short time periods, on the order of a few seconds. For longer times, the errors in the calculated displacements, resulting from uncertain initial conditions and transducer error, were unacceptably large and increased with longer integration times.

In a separate study, accelerometer data were used to obtain modal characteristics of certain bridges for use in damage detection. This study investigated the development of cracks using modal sensitivity (Biswa, Pandey & Samman, 1990). In a related study, the change in dynamic properties was used as a basis for damage detection in prestressed bridges (Flesch & Kernbichler, 1990). The bridges used in the study were instrumented with velocity transducers, with the displacement response obtained by integrating the velocity records.

In a study performed by Purdue University and the Indiana State Highway Commission, bridges were instrumented with accelerometers and displacement transducers to study human perception and comfort with regard to bridge motion (Gaunt & Sutton, 1981). Emphasis was placed on the need for applying baseline corrections to the acceleration response to permit effective integration to obtain the displacement response. The actual corrections were reported in an earlier publication (Kropp, 1977). The results from these studies indicated varying degrees of success for the calculation of the displacement response. The true trace of the displacement response was not reproduced consistently, but calculation of the maximum displacements seemed adequate.

Quick-release experiments have also been used to identify dynamic properties of bridge structures (Douglas, Maragakis & Nath, 1990). A quick-release experiment involves initially displacing a structure using hydraulic rams or cables. The structure is then instantaneously released from that position and allowed to vibrate freely from the initial displacement. The advantage of this method is that the initial conditions are known for each experiment, i.e., zero velocity and known displacement. Douglas et al. (1990) performed this type of experiment and developed a correction algorithm to be used when integrating accelerations to obtain displacements. This algorithm used an nth degree polynomial to fit the acceleration curve and correct for any baseline shift or rotation.
The most extensive application of integrating measured accelerations to calculate displacements has been in earthquake-related research. Data processing of accelerograms has been performed extensively by the Earthquake Engineering Research Laboratory of the California Institute of Technology since the late 1960s (Hudson, Brady & Trifunac, 1969). Most of the structural response information is obtained directly from the accelerogram signal in either the time or frequency domain. The displacement response is calculated by integrating the accelerogram twice, and these techniques are very successful.

The success in calculating the displacement response from the acceleration response in earthquake studies provides a strong basis for developing a similar procedure for bridge vibration studies, although there are a number of differences. Although the response of a structure to an earthquake is complex, there is little low-frequency content in the excitation. Also, the initial and ending conditions are usually known, and the average acceleration and displacement responses are close to zero. In the dynamic response of a bridge, however, there are two distinct phases of motion. First, there is the usual dynamic response composed of a series of higher frequencies. This is superimposed on a pseudo-static response while a vehicle is on the bridge, which is low frequency and essentially time independent. This low-frequency pseudo-static response makes the bridge vibration case unique and the calculation of displacements from accelerations much more difficult. Thus, the focus of this study was the development and evaluation of procedures for determining the displacement response from measured accelerations.

PURPOSE AND SCOPE

The purpose of this research was to develop and evaluate procedures for determining the displacement response of a bridge from measured acceleration data.

The objectives included the following:

- Evaluate and adopt the most appropriate numerical integration procedure to be used with dynamic bridge acceleration data.

- Develop a computer code or adopt an existing software package capable of implementing the numerical integration algorithm selected.

- Plan and implement a field test procedure that would yield experimental acceleration and displacement data recorded at the same points.

- Identify possible errors in the data, and develop correction procedures for minimizing these errors by comparing the measured and calculated displacement responses.
Because of time and resource constraints and cost, field testing was limited to one bridge for which access was possible. However, since the structure selected had dynamic characteristics typical of many slab and girder bridges, this was not considered a serious limitation. Although a variety of acceleration and displacement transducers is available, the experimental data recorded during the field test were obtained using two specific accelerometers and one displacement transducer. The transducers employed can have an effect on the errors in the recorded data, but they had been used successfully in previous field tests and care was taken to minimize possible error introduction.

**METHODOLOGY**

The research consisted of four phases:

1. Review and evaluate various numerical integration procedures from among a variety of integration algorithms proposed and used in previous research applications.

2. Select the numerical integration algorithm most appropriate for application to bridge acceleration data and develop or adopt a computational procedure for implementing the algorithm.

3. Evaluate the accuracy and efficiency of the integration procedure for calculating displacements when applied to simulated and actual acceleration signals. This process included incorporating corrections to the data to account for unknown initial conditions and various errors in the original signal.

4. Apply the refined numerical integration procedure to acceleration data recorded from an actual field test and the represented response from real traffic excitation. The integrated acceleration signals were compared with corresponding measured displacement signals, and final refinements applied.

**Numerical Integration**

The application of numerical integration is used in two applications, as either an approximate procedure for determining the area under a curve of a function, \( f(t) \), or the solution of a differential equation. Although the dynamic response of a simple bridge model may be represented by a differential equation, the complexity of an actual bridge structure makes it impossible to define such an equation. Although numerical integration schemes employed in both of these application areas were considered, the format of the recorded data lent itself more readily to the direct integration of a response function, \( f(t) \).
Numerical integration, used explicitly to evaluate the area underneath a curve represented by a function, \( f(t) \), is frequently referred to as numerical quadrature. The basic procedure employed is to replace the prescribed function, or a set of tabulated data representing the function, by an interpolating polynomial that can then be easily integrated. A variety of quadrature formulas have been developed for performing numerical integration. They differ depending primarily on the particular interpolation polynomial employed.

In this investigation, the response function was represented by a set of data points at equally spaced intervals. Newton-Cotes methods are a class of quadrature formulas specifically designed to handle equally spaced abscissa points and, thus, are particularly appropriate when dealing with tabulated data. These methods use an \( n \)th degree Lagrangian polynomial, which involves only the values of the function and is easily integrable, to replace successive data points.

Factors considered in the selection of the numerical procedure included accuracy, ease of implementation, characteristics of the data, and experiences of previous researchers. After a number of possibilities were evaluated, the Newton-Cotes three-point rule, more commonly known as Simpson's one-third rule, was selected. This algorithm assumes the function to be a second degree curve between three successive points. Its simplicity and accuracy made it advantageous for use with recorded bridge data.

**Computational Procedure**

Two numerical techniques for processing, integrating, and correcting acceleration and velocity signals were considered and evaluated. The first consisted of developing a FORTRAN computer code capable of performing numerical integration using the Newton-Cotes method. The second procedure involved the use of DADiSP, a powerful software tool for the processing, analysis, and display of discrete data records. One of the analysis functions in DADiSP is numerical integration, and the particular algorithm incorporated in the program is Simpson’s one-third rule (DSP Development Corporation, 1991).

The FORTRAN computer program provided considerable flexibility in the analysis of data since the program incorporated a number of numerical algorithms and could be easily modified to meet analysis requirements. It was designed to accept any ASCII data file as input, and the number of data points and spacing of the data can be specified by the user. The use of DADiSP, although limited in the numerical integration procedure available, provided considerable capability for data processing. Not only can any ASCII data with a specified sampling rate be imported into DADiSP, the data can then be processed in a number of ways, including numerical integration, filtering, constant, linear or other corrections, scaling, and display. Plots of the data after processing can be displayed and printed, either for each signal or for combinations of signals using an overplot capability. Another convenient function is the FFT function, which calculates the fast-Fourier transform of a signal and displays the corresponding
frequency content. Accordingly, DADiSP was used for all numerical integration and other processing functions.

**Evaluation of Numerical Integration**

Any acceleration signal recorded during a field test will generally contain complex harmonic and nonperiodic motion. A certain degree of error will be present due to inherent transducer effects and noise generated by electrical connections and wiring. Because of the complexity of errors in measured accelerations, the selected integration and correction procedures to be evaluated were first applied to simpler forms of recorded motion, first on a response that could be expressed analytically and then on data from carefully controlled laboratory experiments. The analytical expressions for acceleration were selected to be simple harmonic functions that were integrated to obtain exact expressions for velocity and displacement. DADiSP was used to discretize the acceleration records, and these discrete records were then integrated numerically. The velocity and displacement records determined from numerical integration were compared to the exact expressions determined analytically so that the numerical integration methods being considered could be evaluated. By the use of analytically generated data, the effect of a number of factors on the final integrated response could be investigated. These included the phase and frequency of the signal components, initial conditions and length of record, correction procedures, and use of digital filters. Faulkner (1993) provides further discussion.

The numerical integration procedures were evaluated further using acceleration data recorded from carefully controlled laboratory experiments, which permitted the application of integration and correction procedures to actual recorded data. The same transducers and data acquisition system used in the field tests were used in these laboratory tests. Transducers included accelerometers, strain gages, and LVDTs. The accelerometers used included force/balance accelerometers, which can record frequencies down to 0 Hz, and high-sensitivity piezoelectric accelerometers with a frequency range from 1 to 5,000 Hz. A data acquisition system was used for recording and analyzing the data, and a signal analyzer was used to display and monitor the output from a transducer. Wolek (1992) provided detailed descriptions concerning the operation of the experimental equipment.

The laboratory tests included a forced vibration test, in which harmonic motion was generated by an electrodynamic shaker, and a test using a simplified model of a three-story frame in which the excitation consisted of release from an initial displacement. These tests were used primarily to evaluate the effect of sample rate and initial conditions on calculated response. The measured accelerations were numerically integrated to obtain displacements, and these calculated displacements were then compared to those measured using the LVDT. A variety of correction procedures were applied to the integrated data to improve the calculated values.
Experimental Field Test

The bridge chosen for the field study was the Dan River Bridge, which carries vehicular traffic on Route 265 over the Dan River. It is located just outside Danville, Virginia, in Pittsylvania County, 2.1 km (1.3 mi) south of U.S. Route 58. The bridge consists of twin bridges, one carrying northbound traffic and the other southbound traffic. At the time of the field test in late 1992, the southbound structure was carrying both north- and southbound traffic and contained the span to be tested. The northbound bridge had been completed but was not yet open for traffic. Both bridges are eight-span structures composed of two identical four-span continuous plate girder sections with concrete decks. Each span is 36.6 m (120 ft) long. An elevation view and a transverse section of the structure are shown in Figures 1 and 2, respectively.

The field test consisted of measuring the accelerations and corresponding displacements and strains of the bridge at selected locations. The measurements were recorded using a series of accelerometers, LVDTs, and strain gages attached to the bottom flanges of the girders. The span selected for instrumentation was controlled to a large extent by availability and ease of access beneath the bridge. Thus, all testing was done on one span of the southbound structure, designated as span B in Figure 1, which was the second span from the south abutment. Scaffolding was erected beneath the span to facilitate instrumentation and provide a fixed reference for the LVDTs. Figure 2 shows a cross-sectional view of the bridge.

NOTE: The figures cited in this report may be found in the Appendix.

A force/balance accelerometer, a high-sensitivity accelerometer, a strain gage, and an LVDT were placed on the bottom flange of each of the five girders at midspan. Figure 3 shows the layout of the instrumentation.

Ten test runs were conducted, and data from the various transducers recorded using the data acquisition system. The data files from these runs are referred to as Files 1 through 10. Recording of the bridge response was initiated when an isolated tractor trailer was approaching the bridge and continued for approximately 2 min. The measured response included the forced vibration phase, while a truck was traversing the bridge, followed by the free vibration phase, when no vehicles were on the structure. With the relatively high volume of traffic on this bridge, the free vibration phase was frequently interrupted by vibration induced by another vehicle entering the span.

Because of the manner in which the recorded data were processed, it was essential that the frequency at which the data were sampled and recorded be at least twice that of the highest frequency of interest. For the Dan River Bridge, frequencies greater than approximately 30 to 50 Hz were expected to contribute little to the overall response. Also, the data acquisition system automatically filtered out frequencies greater than 100 Hz. Accordingly, the sampling rate of the Megadec was set at 200 Hz for the first 6 runs and at 400 Hz for the last 4 runs.
RESULTS AND DISCUSSION

Only selected files from a few of the test runs are discussed in detail since all of the files provided similar response data. Those files selected for detailed evaluation were, for the most part, from midspan transducers and were chosen on the basis of a large-amplitude, well-defined response such as would be produced by an isolated, heavily loaded, tractor trailer. In the discussion of the data analysis, signals consisting of both the forced and free vibration parts of the signal recorded from each accelerometer were integrated, and corrections applied to obtain a representation of displacement response. The correction procedure consisted of subtracting the mean of the response from the acceleration and velocity records before each integration and, where appropriate, applying a highpass filter to the calculated displacement record. The displacements determined from the integration and correction procedures were then compared to the corresponding displacements recorded from the LVDTs. This procedure was followed using first the full recorded accelerometer signal and then applying the procedure separately to the forced and free vibration segments of the full signal.

A typical LVDT displacement record from File 6, resulting from a complete vehicle passage of the bridge, is shown in Figure 4, and the corresponding frequency response spectrum of this signal is shown in Figure 5. From the same test run recorded on File 6, the measured accelerations from the force balance accelerometer and high-sensitivity accelerometer, which were essentially identical, are shown in Figure 6. The corresponding frequency response spectrum of this acceleration signal is given in Figure 7. Qualitatively, the LVDT and accelerometer records, shown in Figures 4 and 6, respectively, were similar.

The frequency spectrum of the LVDT record shown in Figure 5 indicated a strong response at a frequency below 1.0 Hz and a significant response at several frequencies below 5.0 Hz. The frequency spectrum of the corresponding acceleration record, shown in Figure 7, did not show the same response below 1.0 Hz but did show a similar response to the LVDT spectrum above 1.0 Hz. Part of the low-frequency response evident in the LVDT records was likely a result of the pseudo-static response. This was not evident in the spectrum of the acceleration record because the amplitude of that part of the response was very small. However, any response in the acceleration record at frequencies below 1.0 Hz became magnified after integration.

Each recorded signal contained an initial forced vibration phase followed by a period of essentially free vibration, corresponding to the absence of vehicles on the test span. At the beginning of the displacement response shown in Figure 4, the low-frequency response, which includes the peak amplitude, was characteristic of a pseudo-static response. As described earlier, the pseudo-static response is the static component of the total displacement response, and the other dynamic component consists of a series of time-dependent terms. From the frequency spectrum of this displacement record, shown in Figure 5, it was evident that the greatest contribution to the response occurred at around 0.5 Hz; i.e., the amplitude was greatest at that frequency. This frequency was close to the frequency of a pseudo-static response of 0.48 Hz resulting from a typical vehicle traveling at 105 km/hr (65 mph) traversing a simple span bridge.
with a span length of 30.5 m (100 ft). This pseudo-static response provides a significant contribution to the maximum stresses and, therefore, is an essential part of the total displacement response.

A first attempt was made to calculate the displacement response by integrating the full accelerometer signal, such as the one shown in Figure 6. The corresponding frequency response plot is given in Figure 7. The corrections described previously, including filtering, were applied to the accelerometer signals and the resulting velocity and displacement records. The highpass filter applied to the calculated displacement record had a passband frequency of 1.0 Hz. The resulting displacements calculated from the accelerometer record and corrected as described are shown in Figure 8 together with the displacements measured by the corresponding LVDT. For the calculated displacement record, the low-frequency response below 1 Hz was eliminated by application of the highpass filter. This resulted in calculated and measured displacements that compared well during free vibration but did not compare well during the forced vibration phase where the pseudo-static response was significant. Subsequent attempts to retain the pseudo-static part of the response by reducing the cutoff frequency of the highpass filter were unsuccessful.

The acceleration signal shown in Figure 6 was recorded in the field using a sample rate of 200 Hz. To investigate whether a higher sample rate might improve the calculated results, part of the force/balance accelerometer record extracted from File 9, which corresponded to the first vehicle passage, was integrated and corrected. This acceleration signal, shown in Figure 9, was recorded using a sample rate of 400 Hz rather than the 200 Hz used in the File 6 record. The total file consisted of the response from three heavy vehicles crossing the test span one at a time with a spacing of approximately 20 sec. The calculated displacement response, after application of a highpass filter, is shown in Figure 10 with the corresponding LVDT signal. The comparison is very good with an error of approximately 7 percent for the maximum positive displacement. The frequency spectrum of the calculated displacement record also compared well with the frequency spectrum of the measured displacement record as shown in Figure 11. The improved accuracy of the calculated response in this case was attributed to the higher sampling rate.

Similar attempts were made to calculate the displacement response using recorded signals from the high-sensitivity accelerometers. Even with careful error correction and filtering, calculated displacements did not compare favorably with the measured displacements. These results would seem to suggest that the force/balance accelerometer, because of its ability to record very low frequencies, is a superior instrument for recording bridge response.

To confirm the results, a second force/balance accelerometer record, produced from the second vehicle passage of File 9 and shown in Figure 12, was integrated and corrected. A highpass filter was then applied to this calculated displacement signal. The resulting corrected signal, together with the corresponding LVDT signal, is shown in Figure 13. The comparison was very good, with an error of roughly 8 percent for the peak displacement. The frequency spectra of the records shown in Figure 13 are presented in Figure 14; these also compared
favorably. The accuracy of the calculated response from the File 9 records was much greater than that obtained from File 6, again likely because of the higher sampling rate.

In view of the apparent sensitivity of the numerical integration procedure to sample rate and the resulting low-frequency errors encountered in integrating the full acceleration signal, the record was separated into the free and forced vibration portions and the integration and correction procedures were applied to these acceleration segments separately. The first example is the full LVDT signal from File 6, shown in Figure 4. The free vibration portion of this signal was extracted, and the frequency spectrum of this free vibration response is shown in Figure 15. As indicated in the figure, this portion of the signal does not contain a strong response below 1.0 Hz, which would suggest that the pseudo-static response was not present in this part of the record.

The corresponding free vibration part of the force/balance accelerometer signal from File 6 is shown in Figure 16. The displacement signal calculated from this portion of the signal, after application of a highpass filter, is shown in Figure 17 with the corresponding LVDT signal. The comparison is quite good, with a maximum error of less than 10 percent. In this example, in the absence of the low-frequency pseudo-static response, the sample rate of 200 Hz was apparently sufficient to yield reasonably accurate calculated displacements. For the same free vibration record from File 6, the accelerations recorded from the high-sensitivity accelerometer were integrated and corrected in a similar manner using the same highpass filter. The displacement signal calculated from this high-sensitivity accelerometer also compared favorably with the corresponding LVDT signal, with an error of approximately 10 percent.

The displacements calculated from the free vibration part of both the force/balance and high-sensitivity accelerometer records from File 6, recorded at 200 Hz, were significantly more accurate than those obtained using the full signal. The high-sensitivity accelerometer, in particular, did not seem to record low frequencies accurately, and, thus, the results improved significantly once the low frequencies were eliminated. The fact that displacements calculated from these accelerometer records compared favorably with measured displacements would seem to suggest that the 200 Hz sample rate was sufficient to record the free vibration response accurately.

The free vibration response from the force/balance accelerometer signal from File 8, which had a 400 Hz sample rate, is shown in Figure 18 and was analyzed next. The displacement signal after integrating and correcting the accelerometer record is shown in Figure 19 together with the corresponding LVDT signal. The highpass filter used in this example had a passband frequency of 1.0 Hz. The comparison between the calculated and measured displacement signals was quite good, with an error of less than 7 percent. The frequency spectra of the calculated and measured displacement signals of Figure 19, given in Figure 20, also compared well except for a small error around 1.0 Hz. This error may have been due to the choice of the passband frequency of the filter.
Although the results indicated that accurate displacements can be obtained by integration and correction of only the free vibration response, the displacements of usual interest are the maximum values, which generally occur during the forced vibration phase. As a final step in evaluating procedures for calculating displacements from portions of accelerometer records, the forced vibration segments of the acceleration records were integrated and corrected. The forced vibration part of the total accelerometer signal from File 6, measured with a force/balance accelerometer at a sample rate of 200 Hz, was extracted and is shown in Figure 21. This part of the accelerometer signal was initially integrated and corrected using a highpass filter. The resulting calculated displacement is shown in Figure 22 together with the corresponding LVDT signal. As may be seen from the figure, there was a significant difference between the calculated and measured displacement signals. Significant errors were also observed when the frequency response spectra from the calculated and measured displacements were compared. For completeness, the forced vibration acceleration response from the high-sensitivity accelerometer was also integrated and corrected. When these calculated displacements were compared with the LVDT measurements, significant errors were present, as might be expected.

In an attempt to improve the accuracy of the calculated displacements at a low sample rate in the forced vibration regime, a smaller portion of the forced vibration signal was used. This reduced portion, between approximately 31.4 and 33 sec, contained the maximum displacement and was integrated and corrected using a highpass filter. The resulting calculated displacements are shown in Figure 23 with the corresponding portion of the LVDT record. The resulting accuracy of the calculated displacements was quite good, and the error for the maximum displacement was reduced to approximately 1 percent. Thus, if a sufficiently small segment of an acceleration record is considered, it would appear that reliable measures of displacement can be calculated by appropriate integration and correction of the signal.

To evaluate the effect of sample rate further, the forced vibration response of the force/balance accelerometer signal recorded at a sample rate of 400 Hz was integrated and corrected next. The resulting calculated displacement signal is shown in Figure 24 with the corresponding portion of the LVDT signal. The comparison was quite good, with an error in maximum displacement of less than 3 percent. These results again seem to confirm that the higher sample rate of 400 Hz is sufficient to mitigate many of the low-frequency errors and permits an accurate determination of displacements from any segment of an acceleration record.

A careful evaluation of the results from the previous examples indicated that the procedure developed for calculating displacements from measured accelerations can yield reliable answers but considerable care must be exercised in the selection of transducers, the testing procedure, and test parameters such as sample rate. Based on the results, it would appear that the frequency response range of the force/balance accelerometer signals makes them more suitable for recording accelerations from field tests of bridges. The pseudo-static response appears to be an important response component when calculating displacements from acceleration records. The absence of this component from the free vibration segment of an acceleration record makes it possible to calculate displacements from acceleration measurements.
that are quite accurate. The calculated displacements are less accurate when the forced vibration segment is present in the acceleration record. However, even in these latter cases, the use of a high sampling rate on the order of 400 Hz did permit the calculation of reasonably accurate displacements. The success of this numerical integration procedure for determining reliable displacements from measured accelerations would seem to indicate this procedure should be considered when bridge displacements are desired.

CONCLUSIONS

• The numerical integration procedure employed was not a major influence on the success of obtaining reliable displacements from measured accelerations.

• The primary factors affecting the success of the numerical integration procedures were the correction procedures included. For the measured acceleration records obtained from field tests, correction procedures were necessary to account for unknown initial conditions and low-frequency errors due primarily to instrumentation noise. The effect of unknown initial conditions was addressed satisfactorily by subtracting the mean of each signal before integration. The correction procedures adopted to address low-frequency errors posed more difficult problems. The errors in the measured records were generally in the range of less than 1 Hz and most likely due to noise. They caused particular problems because they were amplified significantly after integration. They could be minimized in most cases by applying a highpass filter.

• The LVDT response had low-frequency content not always captured by the accelerometers. This was most likely due to the pseudo-static response, which would not be as evident in acceleration measurements. This response was a significant and essential component of the total displacement response. The force/balance accelerometers were superior to others in their capability to measure and record in the low-frequency range.

• The total response consisted of dynamic response components superimposed on an apparent pseudo-static response. This pseudo-static component was apparent only during the forced vibration phase of the signal corresponding to the presence of a vehicle on the bridge.

• When displacements were calculated by integration and correction of a full accelerometer signal recorded at 400 Hz, the comparison between the calculated displacements and corresponding displacements measured by the LVDTs was very good. Thus, it appears that the sampling rate is a critical factor when recording accelerations that are to be integrated numerically. Further research in this area is indicated.

• The relatively poor comparison between measured displacements and displacements calculated from the full acceleration signal recorded at 200 Hz can be attributed to the presence of the pseudo-static response.
• Displacements calculated from accelerations recorded at a sample rate of 200 Hz differed considerably from the corresponding measured displacements. However, if a sufficiently small portion of the forced vibration phase record was used, rather than the complete forced vibration portion, calculated displacements could be much more accurately determined. However, there was no consistent guideline for choosing the appropriate segment.

• Displacements calculated from the forced vibration phase of acceleration records recorded at a sample rate of 400 Hz were quite accurate. Thus, use of a high sample rate in recording accelerations appears to be the most reliable method of ensuring reasonable accuracy in subsequent calculated displacements.

• Reliable displacements can be calculated from acceleration measurements using numerical integration schemes in conjunction with appropriate correction procedures. In particular, the use of highpass filters with a relatively low cutoff frequency is essential in eliminating low-frequency noise. Also, the use of sample rates of at least 400 Hz in recording accelerations is necessary in mitigating the effect of the pseudo-static response, which is always present in field test data.

• The procedures developed in this investigation for determining displacements from measured accelerations should be employed in future field tests to validate and refine this numerical procedure further and develop specific guidelines for its applicability.

**RECOMMENDATIONS**

The results from this investigation showed that displacements of a bridge structure can be calculated with reasonable accuracy from recorded accelerations, provided proper attention is given to the test procedure, the transducers used to record the accelerations, and, in particular, the test parameters such as sample rate.

Thus, when ambient displacements of a bridge are desired and it is impractical to record displacements experimentally, it is recommended that the numerical integration procedure described in this report be considered as a viable means of calculating displacements from measured accelerations. When this procedure is adopted, it is suggested that force/balance accelerometers be used to record accelerations and that a minimum sample rate of 400 samples per second be employed. Careful consideration should also be given to proper use of digital filters for removing extraneous errors from the signals.
REFERENCES


Appendix

FIGURES
Fig. 1 Elevation View of Bridge. The bridge is symmetric about pier 4 and discontinuous over pier 4.

Fig. 2 Cross-Sectional View of Bridge.
INSTRUMENTATION LAYOUT

Pier 1          Midspan          Pier 2

☐ LVDT, High-Sensitivity and Force/Balance Accelerometers

Fig. 3 Instrumentation Layout for Field Test.
Fig. 4  LVDT Displacement Signal from File 6.

Fig. 5  Frequency Spectrum of LVDT Signal from File 6.
Fig. 6  Force/Balance Acceleration Signal from File 6.

Fig. 7  Frequency Spectrum of Acceleration Record Given in Figure 6.
Fig. 8  Comparison of Calculated Displacement Signal from Force/Balance Accelerometer Record of File 6 to Corresponding Measured Displacement Record. Highpass filter used. Passband = 1.0 Hz and stopband = 0.2 Hz.

Fig. 9  Force/Balance Accelerometer Signal from First Vehicle Passage of File 9.
Fig. 10  Comparison of Highpass Filtered Displacement Signal Calculated from First Vehicle Passage, Force/Balance Accelerometer Record of File 9, to Corresponding Measured Displacement Record. Passband = 0.3 Hz and stopband = 0.1 Hz.

Fig. 11  Frequency Spectrum Comparison of Displacement Signals Given in Figure 10.
Fig. 12  Force/Balance Accelerometer Record from Second Vehicle Passage in File 9.

Fig. 13  Comparision of Highpass Filtered Displacement Signal Calculated from Second Vehicle Passage Force/Balance Accelerometer Record of File 9 to Corresponding Measured Displacement Record. Passband = 0.3 Hz and stopband = 0.1 Hz.
Fig. 14  Frequency Spectrum Comparison of Displacement Signals Given in Figure 13.

Fig. 15  Frequency Spectrum of Free Vibration Part of LVDT Signal from File 6.
Fig. 16  Free Vibratinoon Part of Force/Balance Acceleration Signal from File 6.

Fig. 17  Comparison of Highpass Filtered Displacements Calculated from Free Vibration Force/Balance Acceleration Signal of File 6 to Corresponding LVDT Signal. Passband = 0.1 Hz and stopband = 0.2 Hz.
Fig. 18  Force/Balance Acceleration Signal from File 8.

Fig. 19  Comparison of Highpass Filtered Displacements Calculated from Free Vibration Force/Balance Acceleration Signal from File 8 to Corresponding LVDT Signal. Passband = 0.1 Hz and stopband = 0.8 Hz.
Fig. 20 Frequency Spectrum Comparision of Displacement Signals Given in Figure 19.

Fig. 21 Forced Vibration Part of Force/Balance Acceleration Signal from File 6.
Fig. 22  Comparision of Highpass Filtered Displacement Signal Calculated from Force/Balance Forced Vibration Part of Signal from File 6 to Corresponding LVDT Signal. Passband = 0.3 Hz and stopband = 0.1 Hz.

Fig. 23  Comparision of Highpass Filtered Displacement Signal Calculated from Part of Force/Balance Forced Vibration Signal from File 6 to Corresponding LVDT Signal. Passband = 0.1 Hz and stopband = 0.0 Hz.
Fig. 24  Comparison of Displacements Calculated from Forced Vibration Part of Force/Balance Acceleration Signal from First Vehicle Passage in File 9 to Corresponding LVDT Signal. No filter was used.